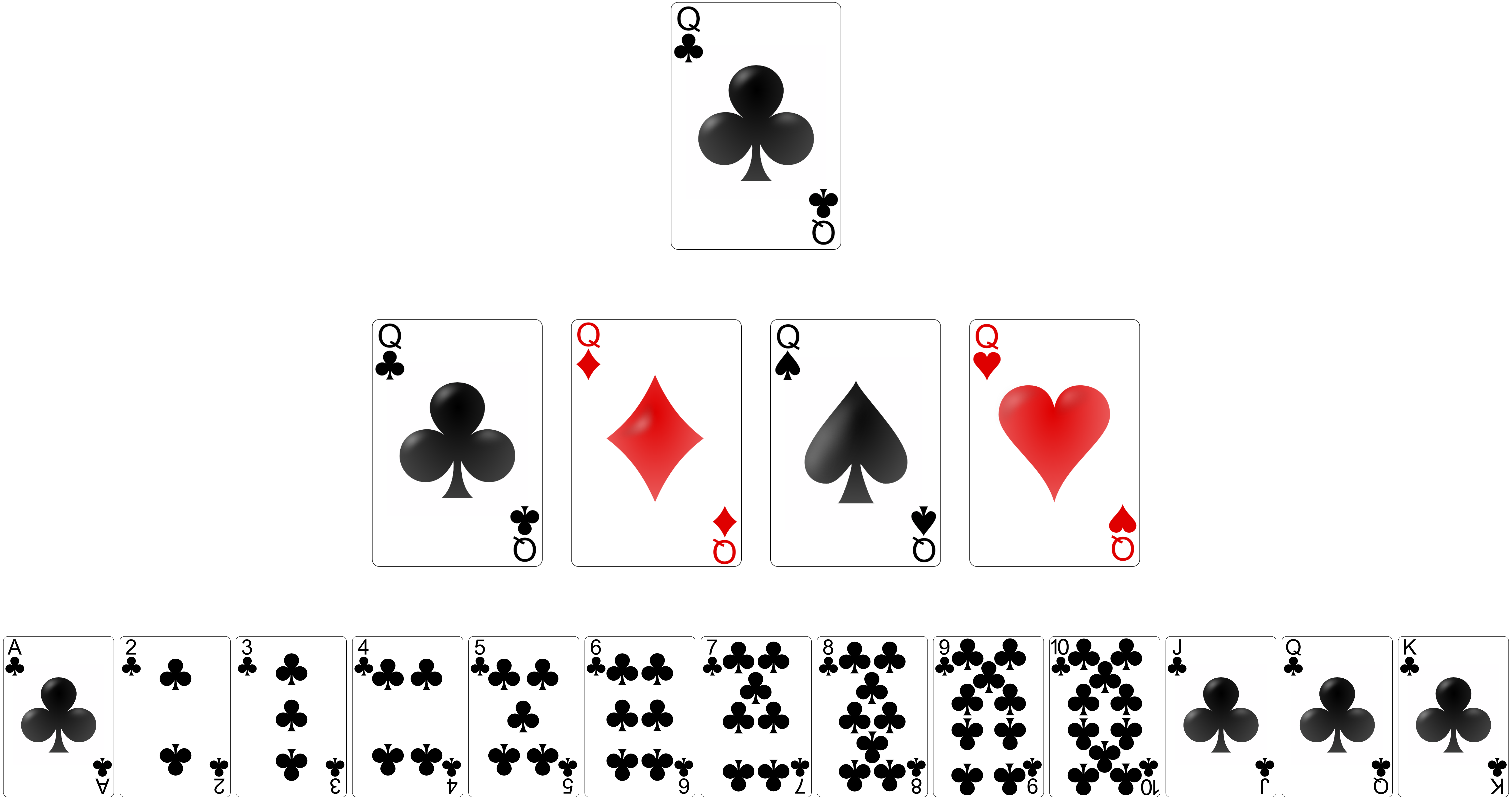
Lessons 009 & 010 Independence and Random Variables

Friday, September 29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Independence

- We saw that sometimes knowing B impacts $P(A)$.
- When this does not occur, we say that A and B are **independent.**
- Specifically, we write $A \perp B$ if

$$P(A \cap B) = P(A)P(B)$$

A system requires two components to function. The components function independently. If component 1 functions 90% of the time, and component 2 functions 85% of the time, what is the probability that the system functions?

0.85	0%
0.90	0%
$0.90 \times 0.85 = 0.765$	0%
$\frac{0.85}{0.9+0.85} = 0.4857$	0%

Independence

- If A and B are **not independent** we say that they are **dependent.**
- If $A \perp B$ then $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$.
- Intuitively, knowledge of A tells us nothing about B and vice-versa.

- A company manufactures both washing machines and dryers.
- 30% of washing machines need service under warranty.
- 10% of dryers need service under warranty.
- **If a customer purchasing both a washer and dryer from this company, what is the probability that they both need service under the warranty, assuming independence?**

$$P(W \cap D) = P(W)P(D) = (0.3)(0.1) = 0.03$$

Independence Properties

- If $A \perp B$ then
  - $B \perp A$
  - $A^C \perp B$
  - $A \perp B^C$
  - $A^C \perp B^C$

- A company manufactures both washing machines and dryers.
- 30% of washing machines need service under warranty.
- 10% of dryers need service under warranty.
- **If a customer purchasing both a washer and dryer from this company, what is the probability that neither need service under warranty?**

$$P(W^C \cap D^C) = P(W^C)P(D^C) = (0.7)(0.9) = 0.63$$

- A company manufactures both washing machines and dryers.
- 30% of washing machines need service under warranty.
- 10% of dryers need service under warranty.
- **If a customer purchasing both a washer and dryer from this company, what is the probability that exactly one will need service under warranty?**

$$
\begin{aligned}
P(W^C \cap D)+P(W \cap D^C) & \\
&= P(W^C)P(D) + P(W)P(D^C) \\
&= (0.7)(0.1) + (0.3)(0.9) \\
&= 0.07 + 0.27 = 0.34
\end{aligned}
$$

30% of students in a class say that pink is their favourite colour. 15% of students say pink is their favourite colour and their favourite number is 7. If these preferences are independent, what proportion of students have 7 as their favourite number?

15%	0%
50%	0%
30%	0%
Not Enough Information	0%

Suppose that a coin is flipped 5 times. Define the following events: A that a tail shows up on flip 1, B that a head shows up on flip 2, C that three heads appear. What statements is true?

$A \perp B$ and $A \perp C$ and $B \perp C$

0%

$A \not\perp B$ and $A \perp C$ and $B \perp C$

0%

$A \perp B$ and $A \not\perp C$ and $B \not\perp C$

0%

$A \not\perp B$ and $A \perp C$ and $B \not\perp C$

0%

A system has two components, but needs only one to function. The components function independently. If component 1 functions 90% of the time, and component 2 functions 85% of the time, what is the probability that the system functions?

$1-(0.1)(0.15)=0.985$	0%
$(0.9)(0.85)+0.9+0.85=2.515$ so probability is 1	0%
$1-0.9\times 0.85=0.235$	0%
$0.9\times 0.85=0.765$	0%

Mutual Independence

- For a set of n events, $A_1, \cdots, A_n$, we say that these events are **mutually independent** if **every subset** (i_k) of **every size** (k) we have

$$P(A_{i_1} \cap \cdots \cap A_{i_k}) = \prod_{j=1}^{k} P(A_{i_j})$$

- A roulette wheel has 18 red, 18 black, and 2 green spaces.
- **What is the probability that in the next three plays on the wheel we observe Green, Black, Red in order?**

$$
\begin{aligned}
&P(\{S_1 = G\} \cap \{S_2 = B\} \cap \{S_3 = R\}) \\
&= P(\{S_1 = G\})P(\{S_2 = B\})P(\{S_3 = R\}) \\
&= \frac{2}{38} \cdot \frac{18}{38} \cdot \frac{18}{38} \\
&= 0.0118093016
\end{aligned}
$$

Mutually Exclusive Events

- If two events **cannot** both happen they are said to be **mutually exclusive.**
- Mutually exclusive events are **always dependent.**

Random Variables

- Often we summarize the results of an experiment with a measure of interest.
    - Instead of sequences of H and T, we report the count of heads.
- If we associated a mapping from events in the sample space $\mathcal{S}$ to the real numbers, this mapping is called a **random variable**.
    - A random variable is a **quantitative variable** that depends on chance.
- We use capital letters to denote random variables, and lower case letters to denote observed outcomes.

Random Variable Examples

- Let X be the number of tails in ten tosses of a coin.
  - $X(\omega) \in \{0,\cdots,10\} \quad \omega \in \{(H, H, H, \cdots, H), \cdots, (T, T, T, \cdots, T)\}$
- Let Z be the sum of three rolls of a die.
  - $Z(\omega) \in \{3,4,\cdots,18\} \quad \omega \in \{(1,1,1), (1,1,2), \cdots, (6,6,6)\}$.
- Let W be the number of days that it rained in a given week.
  - $W(\omega) \in \{0,1,\cdots,7\} \quad \omega \in \{(\text{NR}, \cdots, \text{NR}), \cdots, (\text{R}, \cdots, \text{R})\}$
- Let T be the time that an integrated circuit operates for before failure.

Suppose a student is calling a help desk for support. If the student reaches someone this is a success (S) and otherwise a failure (F). Which of the following is a random variable for this experiment?

$X = S$ if it is a success. 0%

$X = 1$ 0%

$X(S) = 0$ and $X(F) = 1$. 0%

$X(S) = 1$. 0%

An experiment looks at the number of pumps in use at two gas stations, A and B. Take X to be a RV counting the total at $A+B$, Y to be the difference $A-B$, and Z to be the max of A and B. Suppose $x=5, y=-1, z=3$.

$\omega=(5,0)$ 0%

$\omega=(3,3)$ 0%

$\omega=(3,2)$ 0%

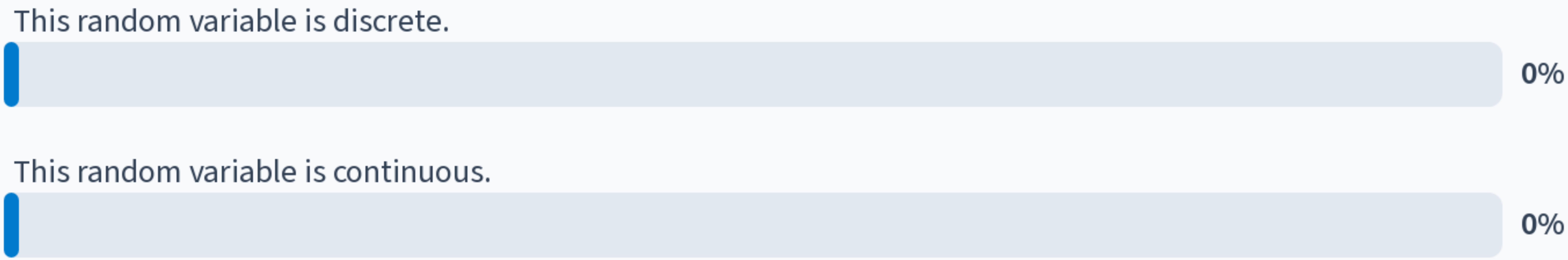
$\omega=(2,3)$ 0%

Example

Discrete versus Continuous

- If you can enumerate all values for a random variable it is **discrete.**
  - Events take the form $\{X = c\}$ or $\{X \in \{c_1, \ldots, c_n\}\}$.
- If you cannot (values taken from sets of intervals), the random variable is **continuous**.
  - Events take the form $\{X \geq c\}$, $\{X \leq c\}$, or $\{X \in [a_1, b_1] \cup \cdots \cup [a_n, b_n]\}$

A random variable takes a value of 1 if a student successfully reaches the help desk, and is 0 otherwise.



This random variable is discrete.

0%

This random variable is continuous.

0%

Batteries coming off of a production line are tested until one fails to meet specification. The total number of trials is recorded as a random variable.

This random variable is discrete.

0%

This random variable is continuous.

0%

A randomly selected piece of carry-on luggage is weighed.

This random variable is discrete. 0%

This random variable is continuous. 0%

The time until an integrated circuit fails is recorded as a random variable T.

This random variable is discrete.

0%

This random variable is continuous.

0%